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COMBINATORIAL STRUCTURE OF PARALLEL AND SERIES CONNECTIONS IN MIN-MAX NEUTROSOPHIC GRAPHS WITH APPLICATIONS TO NETWORK MODELING

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Abstract. This paper introduces the concepts of parallel and series connections in Min-max Neutrosophic Graphs and studies their fundamental properties. The proposed operations are described with illustrative examples. In addition, a new class of neutrosophic graphs, called Max-max Neutrosophic Graphs, is defined and analyzed. It is proved that the number of edges in the parallel connection is twice the product of the numbers of edges of the given graphs, whereas for the series connection it is four times the product. The applicability of the proposed model to electrical circuit networks is also discussed.

1. Introduction

The theory of fuzzy sets, introduced by Zadeh [19], provides a mathematical framework for representing uncertainty, vagueness, and partial membership. This idea has motivated many extensions and applications in discrete mathematics, particularly in graph theory. Rosenfeld [12] initiated the study of fuzzy graphs, which later developed into a rich area involving structural properties, isomorphisms, operations, products, and special classes of fuzzy graphs [2, 4, 5, 14]. In this setting, graph vertices and edges are assigned membership values, allowing graph models to represent networks whose connections are not merely present or absent, but may occur with different degrees of uncertainty.

Further developments were obtained through intuitionistic fuzzy sets introduced by Atanassov [1]. Intuitionistic fuzzy graphs extend fuzzy graphs by considering both membership and non-membership values. This approach has been studied extensively through definitions, operations, complements, regularity, and graph products [9, 10, 11, 3, 6, 7, 8, 13]. These studies show that fuzzy and intuitionistic fuzzy graph models are useful in describing systems where information is incomplete, imprecise, or uncertain.

Another important generalization is provided by neutrosophic set theory, introduced by Smarandache [18, 16, 17]. Neutrosophic sets represent uncertainty by using three independent components, namely truth-membership, indeterminacy-membership, and falsity-membership. This additional indeterminacy component makes neutrosophic models more flexible than classical fuzzy and intuitionistic fuzzy models, especially for systems containing inconsistent or incomplete information. Based on this concept, several types of neutrosophic graphs have been developed, including single-valued neutrosophic graphs,

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isolated single-valued neutrosophic graphs, bipolar neutrosophic graphs, and other related graph structures [21, 20, 22, 23, 24]. Recent studies have also considered products, irregularity, hierarchy, and applications of neutrosophic graphs [25, 26, 27].

Graph operations play an essential role in constructing larger and more complex networks from smaller ones. In classical graph theory, operations such as union, product, join, and other forms of composition are commonly used to study structural behavior and network design. Similar ideas have also been investigated in fuzzy, intuitionistic fuzzy, and neutrosophic graph settings. However, many network structures arising in real applications are not only based on general graph operations, but also on connection patterns that resemble circuit constructions. In particular, parallel and series connections are fundamental in electrical circuit networks. These two types of connections motivate the development of new graph operations that can describe how uncertain components interact when arranged in parallel or in series.

In this paper, we introduce the concepts of parallel and series connections in Min-max Neutrosophic Graphs. These operations are defined by combining the neutrosophic information of vertices and edges through minimum and maximum-based rules. Several illustrative examples are provided to clarify the construction and behavior of the proposed operations. We prove that if two Min-max Neutrosophic Graphs are connected in parallel, then the number of edges in the resulting graph is twice the product of the numbers of edges of the given graphs. Meanwhile, for the series connection, the number of edges is four times the product of the numbers of edges of the given graphs.

In addition, we introduce a new class of neutrosophic graphs, called Max-max Neutrosophic Graphs. This class is defined by using maximum-based rules for the neutrosophic membership values associated with graph elements. Fundamental properties of this new class are discussed, and its relation to the proposed connection operations is analyzed. Finally, we discuss the applicability of the proposed model to electrical circuit networks. The proposed framework is expected to provide a mathematical tool for representing circuit systems whose components contain degrees of truth, indeterminacy, and falsity.

2. Preliminaries and Definitions

Definition 2.1. *A neutrosophic graph on a nonempty set* $\widehat{\mathsf{V}}$ *is defined as* $\widehat{\mathsf{G}} = (\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi, \widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi)$, *where* $\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi : \widehat{\mathsf{V}} \to [0,1]$ *and* $\widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi : \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \to [0,1]$ *represent the truth-membership, indeterminacy-membership, and falsity-membership functions of vertices and edges, respectively, satisfying*

$$\widehat{\mathsf{T}}_\chi(\flat_1, \flat_2) \leq \min\{\widehat{\mathsf{T}}_\xi(\flat_1), \widehat{\mathsf{T}}_\xi(\flat_2)\},$$
$$\widehat{\mathsf{I}}_\chi(\flat_1, \flat_2) \leq \max\{\widehat{\mathsf{I}}_\xi(\flat_1), \widehat{\mathsf{I}}_\xi(\flat_2)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\flat_1, \flat_2) \leq \max\{\widehat{\mathsf{F}}_\xi(\flat_1), \widehat{\mathsf{F}}_\xi(\flat_2)\}$$

for all $\flat_1, \flat_2 \in \widehat{\mathsf{V}}$.

Definition 2.2. *A neutrosophic graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi, \widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi)$ *with underlying graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ *is said to be complete if*

$$\widehat{\mathsf{T}}_\chi(\flat_1, \flat_2) = \min\{\widehat{\mathsf{T}}_\xi(\flat_1), \widehat{\mathsf{T}}_\xi(\flat_2)\},$$
$$\widehat{\mathsf{I}}_\chi(\flat_1, \flat_2) = \max\{\widehat{\mathsf{I}}_\xi(\flat_1), \widehat{\mathsf{I}}_\xi(\flat_2)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\flat_1, \flat_2) = \max\{\widehat{\mathsf{F}}_\xi(\flat_1), \widehat{\mathsf{F}}_\xi(\flat_2)\}$$

for all $\flat_1, \flat_2 \in \widehat{\mathsf{V}}$.

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Definition 2.3. *A neutrosophic graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi, \widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi)$ *with underlying graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ *is called strong if*

$$\widehat{\mathsf{T}}_\chi(\flat_1, \flat_2) = \min\{\widehat{\mathsf{T}}_\xi(\flat_1), \widehat{\mathsf{T}}_\xi(\flat_2)\},$$

$$\widehat{\mathsf{I}}_\chi(\flat_1, \flat_2) = \max\{\widehat{\mathsf{I}}_\xi(\flat_1), \widehat{\mathsf{I}}_\xi(\flat_2)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\flat_1, \flat_2) = \max\{\widehat{\mathsf{F}}_\xi(\flat_1), \widehat{\mathsf{F}}_\xi(\flat_2)\}$$

for all $(\flat_1, \flat_2) \in E$.

Definition 2.4. *A Min-max Neutrosophic Graph is of the form* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$, *where*

(a) $\widehat{\mathsf{V}} = \{\imath_1, \imath_2, \ldots, \imath_n\}$, *together with mappings* $\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi : \widehat{\mathsf{V}} \to [0,1]$, *where* $\widehat{\mathsf{T}}_\xi(\imath_i)$, $\widehat{\mathsf{I}}_\xi(\imath_i)$, *and* $\widehat{\mathsf{F}}_\xi(\imath_i)$ *denote the truth-membership, indeterminacy-membership, and falsity-membership values of the vertex* $\imath_i$, *respectively, satisfying*

$$0 \le \widehat{\mathsf{T}}_\xi(\imath_i) + \widehat{\mathsf{I}}_\xi(\imath_i) + \widehat{\mathsf{F}}_\xi(\imath_i) \le 3 \quad \textit{for every } \imath_i \in \widehat{\mathsf{V}}.$$

(b) $\widehat{\mathsf{E}} \subseteq \widehat{\mathsf{V}} \times \widehat{\mathsf{V}}$, *with mappings* $\widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi : \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \to [0,1]$ *such that*

$$\widehat{\mathsf{T}}_\chi(\imath_i, \imath_j) \le \min\{\widehat{\mathsf{T}}_\xi(\imath_i), \widehat{\mathsf{T}}_\xi(\imath_j)\},$$

$$\widehat{\mathsf{I}}_\chi(\imath_i, \imath_j) \le \max\{\widehat{\mathsf{I}}_\xi(\imath_i), \widehat{\mathsf{I}}_\xi(\imath_j)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_i, \imath_j) \le \max\{\widehat{\mathsf{F}}_\xi(\imath_i), \widehat{\mathsf{F}}_\xi(\imath_j)\}$$

for every $(\imath_i, \imath_j) \in E$.

Definition 2.5. *A Max-min Neutrosophic Graph is of the form* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$, *where*

(a) $\widehat{\mathsf{V}} = \{\imath_1, \imath_2, \ldots, \imath_n\}$, *together with mappings* $\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi : \widehat{\mathsf{V}} \to [0,1]$, *satisfying*

$$0 \le \widehat{\mathsf{T}}_\xi(\imath_i) + \widehat{\mathsf{I}}_\xi(\imath_i) + \widehat{\mathsf{F}}_\xi(\imath_i) \le 3 \quad \textit{for every } \imath_i \in \widehat{\mathsf{V}}.$$

(b) $\widehat{\mathsf{E}} \subseteq \widehat{\mathsf{V}} \times \widehat{\mathsf{V}}$, *with mappings* $\widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi : \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \to [0,1]$ *such that*

$$\widehat{\mathsf{T}}_\chi(\imath_i, \imath_j) \le \max\{\widehat{\mathsf{T}}_\xi(\imath_i), \widehat{\mathsf{T}}_\xi(\imath_j)\},$$

$$\widehat{\mathsf{I}}_\chi(\imath_i, \imath_j) \le \min\{\widehat{\mathsf{I}}_\xi(\imath_i), \widehat{\mathsf{I}}_\xi(\imath_j)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_i, \imath_j) \le \min\{\widehat{\mathsf{F}}_\xi(\imath_i), \widehat{\mathsf{F}}_\xi(\imath_j)\}$$

for every $(\imath_i, \imath_j) \in E$.

Definition 2.6. *A neutrosophic graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ *is called a complete neutrosophic graph if*

$$\widehat{\mathsf{T}}_\chi(\imath_i, \imath_j) = \min\{\widehat{\mathsf{T}}_\xi(\imath_i), \widehat{\mathsf{T}}_\xi(\imath_j)\},$$

$$\widehat{\mathsf{I}}_\chi(\imath_i, \imath_j) = \max\{\widehat{\mathsf{I}}_\xi(\imath_i), \widehat{\mathsf{I}}_\xi(\imath_j)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_i, \imath_j) = \max\{\widehat{\mathsf{F}}_\xi(\imath_i), \widehat{\mathsf{F}}_\xi(\imath_j)\}$$

for all $\imath_i, \imath_j \in \widehat{\mathsf{V}}$.

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4. Series and Parallel Connections in Neutrosophic Graphs

Let $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ and $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}', \widehat{\mathsf{E}}')$ be two neutrosophic graphs, where

$$\widehat{\mathsf{V}} = \{\imath_1, \imath_2, \imath_3, \ldots, \imath_n\}$$

and

$$\widehat{\mathsf{V}}' = \{\imath'_1, \imath'_2, \imath'_3, \ldots, \imath'_n\},$$

with

$$\widehat{\mathsf{E}} \subseteq \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \quad \text{and} \quad \widehat{\mathsf{E}}' \subseteq \widehat{\mathsf{V}}' \times \widehat{\mathsf{V}}'.$$

Suppose the corresponding truth-membership, indeterminacy-membership, and falsity-membership functions are given by

$$\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi : \widehat{\mathsf{V}} \to [0,1],$$
$$\widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi : \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \to [0,1],$$

and

$$\widehat{\mathsf{T}}'_\xi, \widehat{\mathsf{I}}'_\xi, \widehat{\mathsf{F}}'_\xi : \widehat{\mathsf{V}}' \to [0,1],$$
$$\widehat{\mathsf{T}}'_\chi, \widehat{\mathsf{I}}'_\chi, \widehat{\mathsf{F}}'_\chi : \widehat{\mathsf{V}}' \times \widehat{\mathsf{V}}' \to [0,1].$$

Definition 4.1. *The parallel connection of $\widehat{\mathsf{G}}_1$ and $\widehat{\mathsf{G}}_2$ with respect to the edges e_1 and e_2 in the neutrosophic graphs, denoted by $P((\widehat{\mathsf{G}}_1, \mathsf{e}_1) : (\widehat{\mathsf{G}}_2, \mathsf{e}_2))$, is obtained by deleting the edge $\mathsf{e}_1 \in \widehat{\mathsf{G}}_1$ and $\mathsf{e}_2 \in \widehat{\mathsf{G}}_2$, and identifying the end vertices $\imath_m \in \widehat{\mathsf{G}}_1$ of e_1 and $\imath'_n \in \widehat{\mathsf{G}}_2$ of e_2 as a new vertex $\widehat{\mathsf{V}}$, such that $\widehat{\mathsf{T}}_\xi(v) = \widehat{\mathsf{T}}_\xi(\imath_m) \wedge \widehat{\mathsf{T}}_\xi(\imath'_n)$, $\widehat{\mathsf{I}}_\xi(v) = \widehat{\mathsf{I}}_\xi(\imath_m) \vee \widehat{\mathsf{I}}_\xi(\imath'_n)$, and $\widehat{\mathsf{F}}_\xi(v) = \widehat{\mathsf{F}}_\xi(\imath_m) \vee \widehat{\mathsf{F}}_\xi(\imath'_n)$.*

Similarly, identify the other two end vertices $\imath_n \in \widehat{\mathsf{G}}_1$ of e_1 and $\imath'_m \in \widehat{\mathsf{G}}_2$ of e_2 as a new vertex $\imath'$, such that

$$\widehat{\mathsf{T}}_\xi(\imath') = \widehat{\mathsf{T}}_\xi(\imath_n) \wedge \widehat{\mathsf{T}}_\xi(\imath'_m),$$
$$\widehat{\mathsf{I}}_\xi(\imath') = \widehat{\mathsf{I}}_\xi(\imath_n) \vee \widehat{\mathsf{I}}_\xi(\imath'_m),$$

and

$$\widehat{\mathsf{F}}_\xi(\imath') = \widehat{\mathsf{F}}_\xi(\imath_n) \vee \widehat{\mathsf{F}}_\xi(\imath'_m).$$

Finally, introduce a new edge $\imath\imath'$, such that

$$\widehat{\mathsf{T}}_\chi(\imath\imath') = \widehat{\mathsf{T}}_\chi(\mathsf{e}_1) \wedge \widehat{\mathsf{T}}_\chi(\mathsf{e}_2),$$
$$\widehat{\mathsf{I}}_\chi(\imath\imath') = \widehat{\mathsf{I}}_\chi(\mathsf{e}_1) \vee \widehat{\mathsf{I}}_\chi(\mathsf{e}_2),$$

and

$$\widehat{\mathsf{F}}_\chi(\imath\imath') = \widehat{\mathsf{F}}_\chi(\mathsf{e}_1) \vee \widehat{\mathsf{F}}_\chi(\mathsf{e}_2).$$

Definition 4.2. *The series connection of $\widehat{\mathsf{G}}_1$ and $\widehat{\mathsf{G}}_2$ with respect to the edges e_1 and e_2 in the neutrosophic graphs, denoted by $S((\widehat{\mathsf{G}}_1, \mathsf{e}_1) : (\widehat{\mathsf{G}}_2, \mathsf{e}_2))$, is obtained by deleting the edge $\mathsf{e}_1 \in \widehat{\mathsf{G}}_1$ and $\mathsf{e}_2 \in \widehat{\mathsf{G}}_2$, and identifying one of the end vertices $\imath_m \in \widehat{\mathsf{G}}_1$ of e_1 and $\imath'_n \in \widehat{\mathsf{G}}_2$ of e_2 as a new vertex $\widehat{\mathsf{V}}$, such that*

$$\widehat{\mathsf{T}}_\xi(v) = \widehat{\mathsf{T}}_\xi(\imath_m) \wedge \widehat{\mathsf{T}}_\xi(\imath'_n),$$
$$\widehat{\mathsf{I}}_\xi(v) = \widehat{\mathsf{I}}_\xi(\imath_m) \vee \widehat{\mathsf{I}}_\xi(\imath'_n),$$

and

$$\widehat{\mathsf{F}}_\xi(v) = \widehat{\mathsf{F}}_\xi(\imath_m) \vee \widehat{\mathsf{F}}_\xi(\imath'_n).$$

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Finally, introduce a new edge $\imath_n\imath'_m$, *where* $\imath_n \in \widehat{\mathsf{G}}_1$ *is an end vertex of* e_1 *and* $\imath'_m \in \widehat{\mathsf{G}}_2$ *is an end vertex of* e_2, *such that*

$$\widehat{\mathsf{T}}_\chi(\imath_n\imath'_m) = \widehat{\mathsf{T}}_\chi(\mathsf{e}_1) \wedge \widehat{\mathsf{T}}_\chi(\mathsf{e}_2),$$
$$\widehat{\mathsf{I}}_\chi(\imath_n\imath'_m) = \widehat{\mathsf{I}}_\chi(\mathsf{e}_1) \vee \widehat{\mathsf{I}}_\chi(\mathsf{e}_2),$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_n\imath'_m) = \widehat{\mathsf{F}}_\chi(\mathsf{e}_1) \vee \widehat{\mathsf{F}}_\chi(\mathsf{e}_2).$$

Example 4.1. *Let* $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ *and* $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$ *be two neutrosophic graphs. We, take* $\widehat{\mathsf{V}}_1 = \{a, b, c\}$, $\widehat{\mathsf{V}}_2 = \{p, q, r\}$. *The neutrosophic vertex values are given as follows:*

$$a = (0.7, 0.2, 0.1), \qquad p = (0.6, 0.1, 0.2),$$
$$b = (0.6, 0.3, 0.1), \qquad q = (0.5, 0.2, 0.2),$$
$$c = (0.5, 0.2, 0.2), \qquad r = (0.4, 0.3, 0.2).$$

Suppose $\mathsf{e}_1 = ab \in \widehat{\mathsf{G}}_1$ *and* $\mathsf{e}_2 = pq \in \widehat{\mathsf{G}}_2$ *with edge values* $\mathsf{e}_1 = (0.6, 0.3, 0.1)$, $\mathsf{e}_2 = (0.5, 0.2, 0.2)$.

Parallel Connection: *Identify vertices* a *and* p *as a new vertex* $\widehat{\mathsf{V}}$*:*

$$\widehat{\mathsf{T}}_\xi(v) = 0.7 \wedge 0.6 = 0.6,\ \widehat{\mathsf{I}}_\xi(v) = 0.2 \vee 0.1 = 0.2,\ \text{and}\ \widehat{\mathsf{F}}_\xi(v) = 0.1 \vee 0.2 = 0.2.$$

Similarly, identify b *and* q *as a new vertex* $\imath'$*:*

$$\widehat{\mathsf{T}}_\xi(\imath') = 0.6 \wedge 0.5 = 0.5,\ \widehat{\mathsf{I}}_\xi(\imath') = 0.3 \vee 0.2 = 0.3,\ \text{and}\ \widehat{\mathsf{F}}_\xi(\imath') = 0.1 \vee 0.2 = 0.2.$$

Now, we introduce the new edge $\imath\imath'$*:*

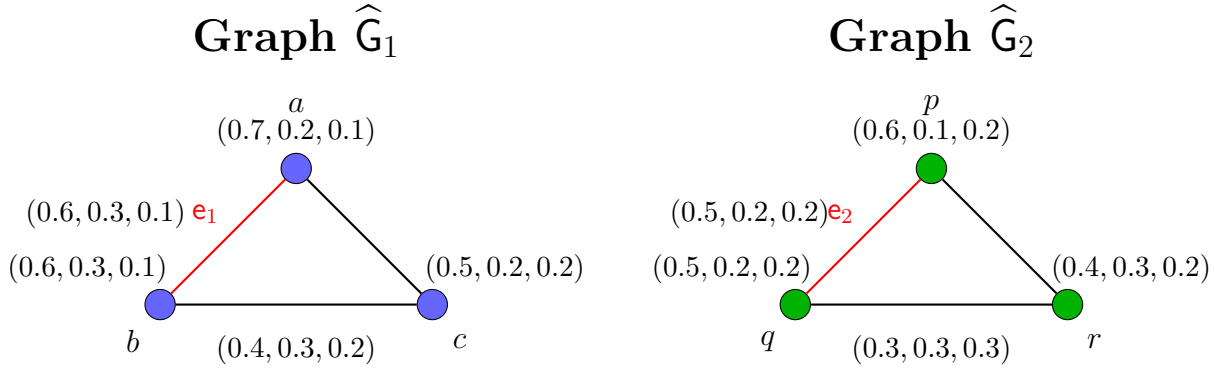
$$\widehat{\mathsf{T}}_\chi(\imath\imath') = 0.6 \wedge 0.5 = 0.5,\ \widehat{\mathsf{I}}_\chi(\imath\imath') = 0.3 \vee 0.2 = 0.3,\ \text{and}\ \widehat{\mathsf{F}}_\chi(\imath\imath') = 0.1 \vee 0.2 = 0.2.$$

Hence, $\imath\imath' = (0.5, 0.3, 0.2)$.

The graphical illustration of the parallel connection of the neutrosophic graphs $\widehat{\mathsf{G}}_1$ and $\widehat{\mathsf{G}}_2$ is shown below. In this construction, the selected edges $\mathsf{e}_1 = ab$ and $\mathsf{e}_2 = pq$ are removed from the respective graphs, and the corresponding vertices are identified to produce new vertices in the resulting graph. The neutrosophic membership values of the newly formed vertices and edges are determined using the min–max operations on the truth, indeterminacy, and falsity memberships. The resulting graph represents the parallel connection $P((\widehat{\mathsf{G}}_1, \mathsf{e}_1) : (\widehat{\mathsf{G}}_2, \mathsf{e}_2))$. For better understanding, the corresponding graphical representation can be seen below.

FIGURE 1 (PARALLEL CONNECTION)

Let $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ and $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$ be two neutrosophic graphs. Let $\widehat{\mathsf{V}}_1 = \{a, b, c\}$, $\widehat{\mathsf{V}}_2 = \{p, q, r\}$. Consider $\mathsf{e}_1 = ab \in \widehat{\mathsf{E}}_1$ and $\mathsf{e}_2 = pq \in \widehat{\mathsf{E}}_2$.



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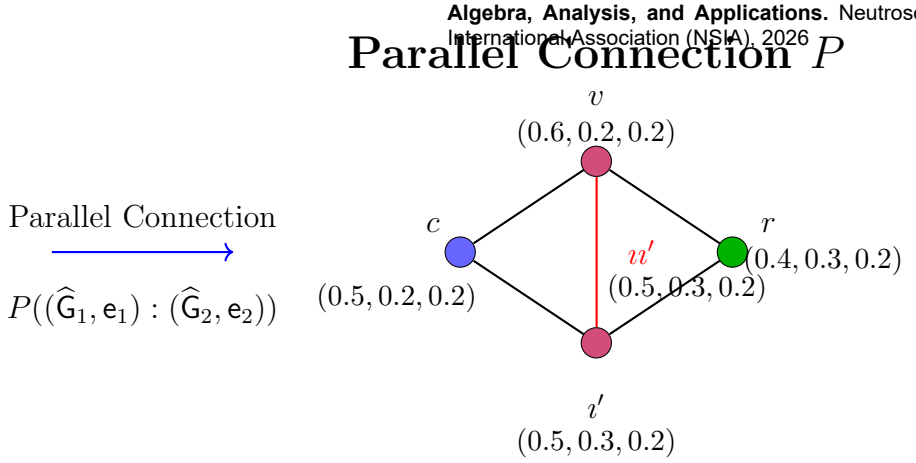
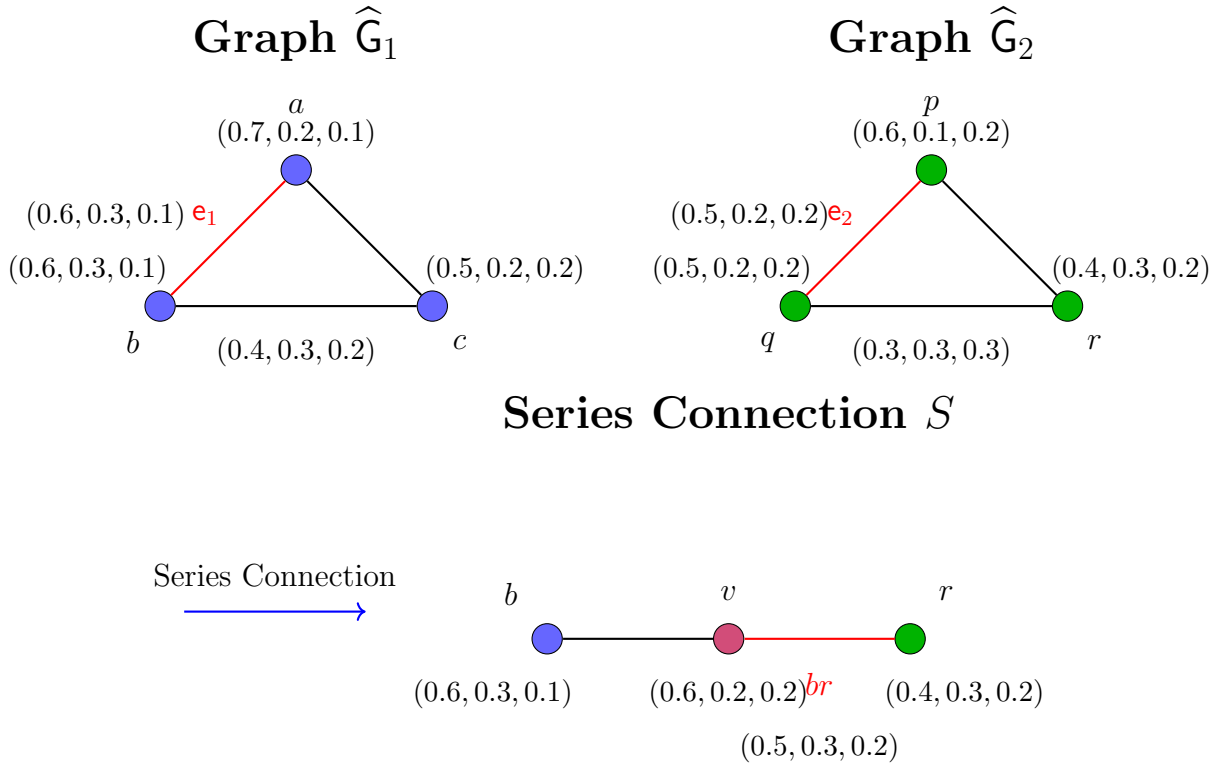


FIGURE 2 (SERIES CONNECTION)



Theorem 4.1. *Let $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ and $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}', \widehat{\mathsf{E}}')$ be two complete Min-max Neutrosophic Graphs. Then, the parallel connection $P(\widehat{\mathsf{G}}_1, \widehat{\mathsf{G}}_2)$ and the series connection $S(\widehat{\mathsf{G}}_1, \widehat{\mathsf{G}}_2)$ need not be complete neutrosophic graphs and are not necessarily Min-max Neutrosophic Graphs.*

The proof follows from the following example.

Example 4.2. *Consider the complete Min-max Neutrosophic cycle graph $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ and the complete Min-max Neutrosophic star graph $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$, shown in the above figures.*

For the cycle graph $\widehat{\mathsf{G}}_1$, let

$$\widehat{\mathsf{V}}_1 = \{a, b, c, d\},$$

with vertex values

$$a = (0.8, 0.1, 0.1), \quad b = (0.7, 0.2, 0.1), \quad c = (0.6, 0.2, 0.2), \quad d = (0.5, 0.3, 0.2).$$

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The highlighted edge is

$$\mathsf{e}_1 = ab = (0.7, 0.2, 0.1).$$

Similarly, for the star graph $\widehat{\mathsf{G}}_2$*, let* $\widehat{\mathsf{V}}_2 = \{p, q, r, s\}$*, with*

$$p = (0.7, 0.1, 0.2), \quad q = (0.6, 0.2, 0.2), \quad r = (0.5, 0.3, 0.2), \quad s = (0.4, 0.3, 0.3).$$

The selected edge is

$$\mathsf{e}_2 = pq = (0.6, 0.2, 0.2).$$

Now, we construct the parallel connection

$$P((\widehat{\mathsf{G}}_1, \mathsf{e}_1) : (\widehat{\mathsf{G}}_2, \mathsf{e}_2)),$$

as illustrated in the corresponding figure. After identifying the corresponding vertices, the new connecting edge becomes

$$\omega\omega' = (0.6, 0.2, 0.2).$$

For a complete Min-max Neutrosophic Graph, each edge must satisfy

$$\widehat{\mathsf{T}}_\xi(\omega\omega') = \min\{\widehat{\mathsf{T}}_\xi(\omega), \widehat{\mathsf{T}}_\xi(\omega')\},$$

$$\widehat{\mathsf{I}}_\xi(\omega\omega') = \max\{\widehat{\mathsf{I}}_\xi(\omega), \widehat{\mathsf{I}}_\xi(\omega')\},$$

and

$$\widehat{\mathsf{F}}_\xi(\omega\omega') = \max\{\widehat{\mathsf{F}}_\xi(\omega), \widehat{\mathsf{F}}_\xi(\omega')\}.$$

From the graph,

$$\omega = (0.7, 0.1, 0.2), \qquad \omega' = (0.6, 0.2, 0.2).$$

Hence,

$$\widehat{\mathsf{T}}_\xi(\omega\omega') = \min\{0.7, 0.6\} = 0.6, \widehat{\mathsf{I}}_\xi(\omega\omega') = \max\{0.1, 0.2\} = 0.2, \widehat{\mathsf{F}}_\xi(\omega\omega') = \max\{0.2, 0.2\} = 0.2.$$

Although the edge $\omega\omega'$ *satisfies the condition, the remaining newly formed edges in the parallel connection do not necessarily preserve the completeness property for all vertex pairs. In particular, some edges are absent between vertices that should be adjacent in a complete graph. Therefore,*

$$P(\widehat{\mathsf{G}}_1, \widehat{\mathsf{G}}_2)$$

is not necessarily a complete Min-max Neutrosophic Graph.

Similarly, in the wheel-type connection graph, new spokes and connecting edges are introduced. These newly created edges may fail to satisfy the Min-max conditions for truth, indeterminacy, and falsity memberships. Hence, the resulting series-type connection also need not remain complete.

Therefore, $P(\widehat{\mathsf{G}}_1, \widehat{\mathsf{G}}_2)$ *and* $S(\widehat{\mathsf{G}}_1, \widehat{\mathsf{G}}_2)$ *are not necessarily complete Min-max Neutrosophic Graphs.*

The corresponding graphical representations are given below for better visualization of the proposed neutrosophic graph structures.

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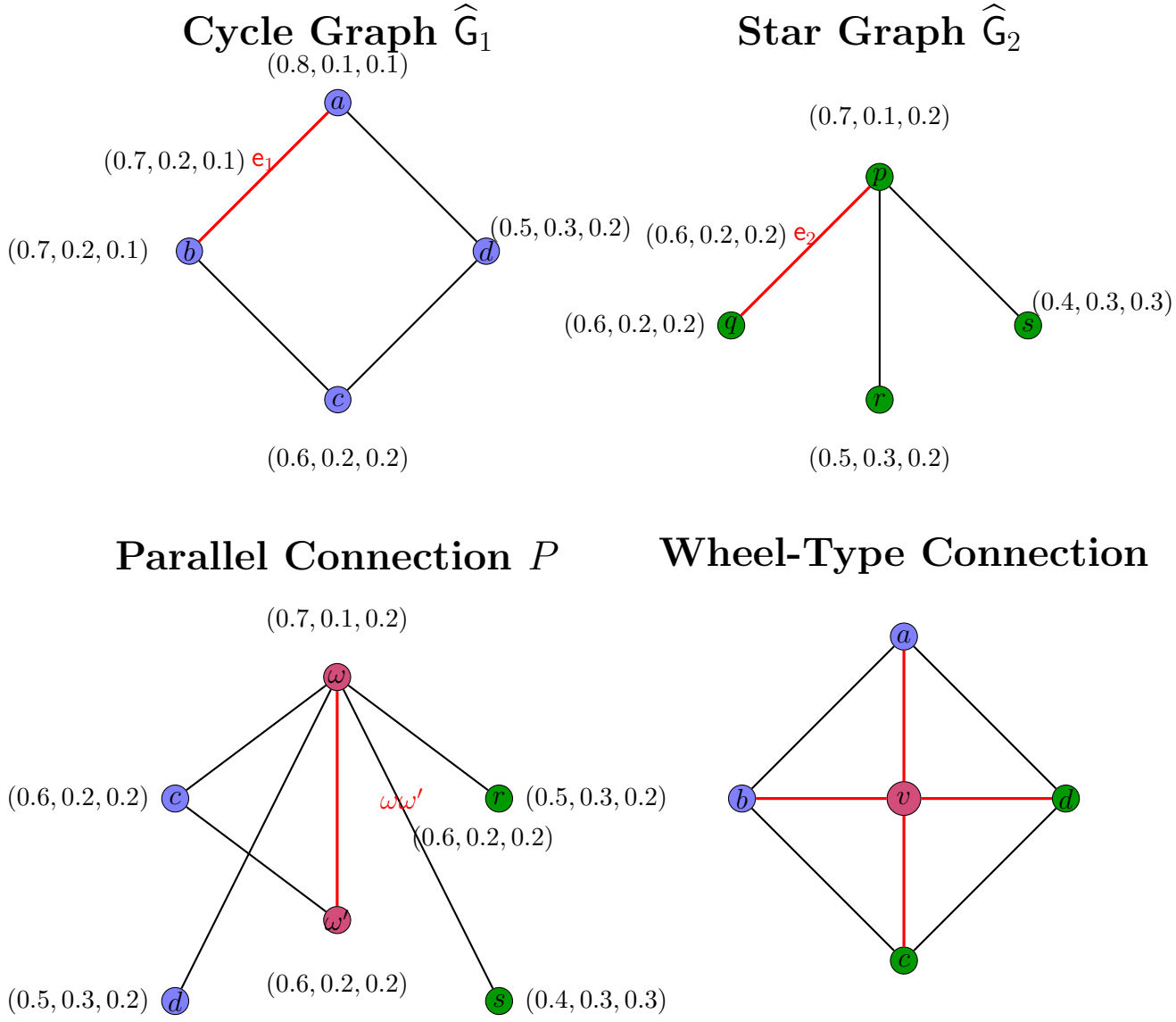

Definition 4.3. *A Max-max Neutrosophic Graph is of the form* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$, *where*

(a) $\widehat{\mathsf{V}} = \{\imath_1, \imath_2, \ldots, \imath_n\}$ *together with mappings* $\widehat{\mathsf{T}}_\xi, \widehat{\mathsf{I}}_\xi, \widehat{\mathsf{F}}_\xi : \widehat{\mathsf{V}} \to [0, 1]$, *where*

$$\widehat{\mathsf{T}}_\xi(\imath_i),\ \widehat{\mathsf{I}}_\xi(\imath_i),\ \widehat{\mathsf{F}}_\xi(\imath_i)$$

denote the truth-membership, indeterminacy-membership, and falsity-membership values of the vertex $\imath_i \in \widehat{\mathsf{V}}$, *respectively, satisfying* $0 \leq \widehat{\mathsf{T}}_\xi(\imath_i) + \widehat{\mathsf{I}}_\xi(\imath_i) + \widehat{\mathsf{F}}_\xi(\imath_i) \leq 3$ *for every* $\imath_i \in \widehat{\mathsf{V}}$, $(i = 1, 2, \ldots, n)$.

(b) $\widehat{\mathsf{E}} \subseteq \widehat{\mathsf{V}} \times \widehat{\mathsf{V}}$, *with mappings* $\widehat{\mathsf{T}}_\chi, \widehat{\mathsf{I}}_\chi, \widehat{\mathsf{F}}_\chi : \widehat{\mathsf{V}} \times \widehat{\mathsf{V}} \to [0, 1]$ *such that*

$$\widehat{\mathsf{T}}_\chi(\imath_i, \imath_j) \leq \max\{\widehat{\mathsf{T}}_\xi(\imath_i), \widehat{\mathsf{T}}_\xi(\imath_j)\},$$

$$\widehat{\mathsf{I}}_\chi(\imath_i, \imath_j) \leq \max\{\widehat{\mathsf{I}}_\xi(\imath_i), \widehat{\mathsf{I}}_\xi(\imath_j)\},$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_i, \imath_j) \leq \max\{\widehat{\mathsf{F}}_\xi(\imath_i), \widehat{\mathsf{F}}_\xi(\imath_j)\}$$

for every $(\imath_i, \imath_j) \in E$.

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Here, the quadruple $(\imath_i, \widehat{\mathsf{T}}_\xi(\imath_i), \widehat{\mathsf{I}}_\xi(\imath_i), \widehat{\mathsf{F}}_\xi(\imath_i))$ *represents the truth-membership, indeterminacy-membership, and falsity-membership values of the vertex* $\imath_i \in \widehat{\mathsf{V}}$. *Similarly, the quadruple*

$$((\imath_i, \imath_j), \widehat{\mathsf{T}}_\chi(\imath_i, \imath_j), \widehat{\mathsf{I}}_\chi(\imath_i, \imath_j), \widehat{\mathsf{F}}_\chi(\imath_i, \imath_j))$$

represents the truth-membership, indeterminacy-membership, and falsity-membership values of the edge $(\imath_i, \imath_j) \in E$.

Example 4.3. *Consider the graph* $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$, *where* $\widehat{\mathsf{V}} = \{\imath_1, \imath_2, \imath_3\}$ *and* $E = \{\imath_1\imath_2, \imath_2\imath_3, \imath_1\imath_3\}$. *Now, we define the neutrosophic vertex values as follows:*

$$\imath_1 = (0.6, 0.2, 0.1),\ \imath_2 = (0.7, 0.1, 0.2),\ \textit{and}\ \imath_3 = (0.5, 0.3, 0.2).$$

The corresponding edge values are given by

$$\imath_1\imath_2 = (0.7, 0.2, 0.2),\ \imath_2\imath_3 = (0.7, 0.3, 0.2),\ \textit{and}\ \imath_1\imath_3 = (0.6, 0.3, 0.2).$$

Now, we verify the Max-max neutrosophic conditions. For the edge $\imath_1\imath_2$,

$$\widehat{\mathsf{T}}_\chi(\imath_1, \imath_2) = 0.7 \leq \max\{0.6, 0.7\} = 0.7,$$

$$\widehat{\mathsf{I}}_\chi(\imath_1, \imath_2) = 0.2 \leq \max\{0.2, 0.1\} = 0.2,$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_1, \imath_2) = 0.2 \leq \max\{0.1, 0.2\} = 0.2.$$

Similarly,

$$\widehat{\mathsf{T}}_\chi(\imath_2, \imath_3) = 0.7 \leq \max\{0.7, 0.5\} = 0.7,$$

$$\widehat{\mathsf{I}}_\chi(\imath_2, \imath_3) = 0.3 \leq \max\{0.1, 0.3\} = 0.3,$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_2, \imath_3) = 0.2 \leq \max\{0.2, 0.2\} = 0.2.$$

Also,

$$\widehat{\mathsf{T}}_\chi(\imath_1, \imath_3) = 0.6 \leq \max\{0.6, 0.5\} = 0.6,$$

$$\widehat{\mathsf{I}}_\chi(\imath_1, \imath_3) = 0.3 \leq \max\{0.2, 0.3\} = 0.3,$$

and

$$\widehat{\mathsf{F}}_\chi(\imath_1, \imath_3) = 0.2 \leq \max\{0.1, 0.2\} = 0.2.$$

Hence, all the required conditions are satisfied. Therefore, $\widehat{\mathsf{G}} = (\widehat{\mathsf{V}}, \widehat{\mathsf{E}})$ *is a Max-max Neutrosophic Graph.*

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From the following graph, we observe the structure and neutrosophic labeling of the Max-max Neutrosophic Graph.

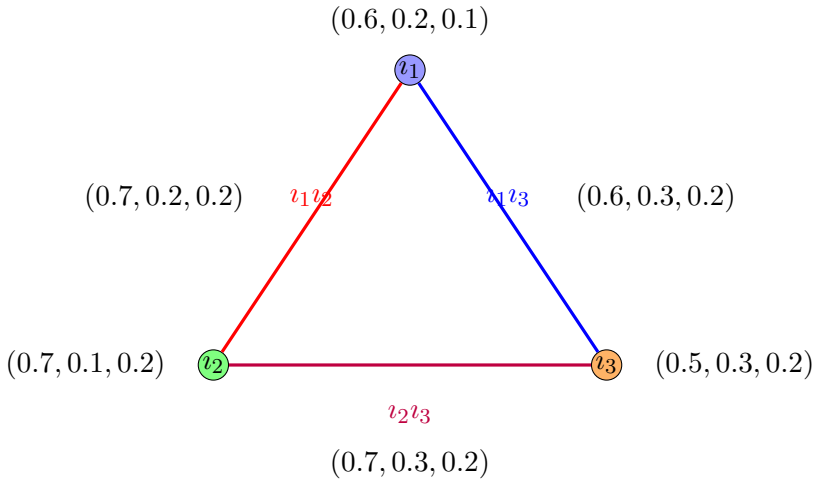


Figure 3. Max-max Neutrosophic Graph

Theorem 4.2. *Let* $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ *and* $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$ *be two Min-max Neutrosophic Graphs having* m *and* n *edges, respectively. Then, the number of possible parallel connections between* $\widehat{\mathsf{G}}_1$ *and* $\widehat{\mathsf{G}}_2$ *is* $2mn$.

Proof. Suppose $|\widehat{\mathsf{E}}_1| = m$ and $|\widehat{\mathsf{E}}_2| = n$. Let $\widehat{\mathsf{E}}_1 = \{\mathsf{e}_1, \mathsf{e}_2, \ldots, \mathsf{e}_m\}$ and $\widehat{\mathsf{E}}_2 = \{\mathsf{e}'_1, \mathsf{e}'_2, \ldots, \mathsf{e}'_n\}$.

Each edge of a neutrosophic graph is associated with truth-membership, indeterminacy-membership, and falsity-membership values. Thus, for every edge $\mathsf{e}_i = u_1u_2 \in \widehat{\mathsf{E}}_1$, there exist neutrosophic edge values $(\widehat{\mathsf{T}}_\chi(\mathsf{e}_i), \widehat{\mathsf{I}}_\chi(\mathsf{e}_i), \widehat{\mathsf{F}}_\chi(\mathsf{e}_i))$, and similarly, for every edge $\mathsf{e}'_j = \imath_1\imath_2 \in \widehat{\mathsf{E}}_2$, there exist $(\widehat{\mathsf{T}}_\chi(\mathsf{e}'_j), \widehat{\mathsf{I}}_\chi(\mathsf{e}'_j), \widehat{\mathsf{F}}_\chi(\mathsf{e}'_j))$. To construct a parallel connection, one edge from $\widehat{\mathsf{G}}_1$ and one edge from $\widehat{\mathsf{G}}_2$ are selected and deleted.

The number of ways to choose one edge from $\widehat{\mathsf{G}}_1$ is m, and the number of ways to choose one edge from $\widehat{\mathsf{G}}_2$ is n. Hence, the total number of possible selections of deleted edge pairs is mn.

Now, consider a fixed pair of deleted edges $\mathsf{e}_i = u_1u_2$ and $\mathsf{e}'_j = \imath_1\imath_2$. By the definition of parallel connection in Min-max Neutrosophic Graphs, the end vertices can be identified in two possible ways.

First identification: $u_1 \leftrightarrow \imath_1$, $u_2 \leftrightarrow \imath_2$. The newly formed vertices satisfy

$$\widehat{\mathsf{T}}_\xi(w_1) = \widehat{\mathsf{T}}_\xi(u_1) \wedge \widehat{\mathsf{T}}_\xi(\imath_1),$$
$$\widehat{\mathsf{I}}_\xi(w_1) = \widehat{\mathsf{I}}_\xi(u_1) \vee \widehat{\mathsf{I}}_\xi(\imath_1),$$
$$\widehat{\mathsf{F}}_\xi(w_1) = \widehat{\mathsf{F}}_\xi(u_1) \vee \widehat{\mathsf{F}}_\xi(\imath_1),$$

and

$$\widehat{\mathsf{T}}_\xi(w_2) = \widehat{\mathsf{T}}_\xi(u_2) \wedge \widehat{\mathsf{T}}_\xi(\imath_2),$$
$$\widehat{\mathsf{I}}_\xi(w_2) = \widehat{\mathsf{I}}_\xi(u_2) \vee \widehat{\mathsf{I}}_\xi(\imath_2),$$
$$\widehat{\mathsf{F}}_\xi(w_2) = \widehat{\mathsf{F}}_\xi(u_2) \vee \widehat{\mathsf{F}}_\xi(\imath_2).$$

The newly introduced edge w_1w_2 has neutrosophic values

$$\widehat{\mathsf{T}}_\chi(w_1w_2) = \widehat{\mathsf{T}}_\chi(\mathsf{e}_i) \wedge \widehat{\mathsf{T}}_\chi(\mathsf{e}'_j),$$
$$\widehat{\mathsf{I}}_\chi(w_1w_2) = \widehat{\mathsf{I}}_\chi(\mathsf{e}_i) \vee \widehat{\mathsf{I}}_\chi(\mathsf{e}'_j),$$

and

$$\widehat{\mathsf{F}}_\chi(w_1w_2) = \widehat{\mathsf{F}}_\chi(\mathsf{e}_i) \vee \widehat{\mathsf{F}}_\chi(\mathsf{e}'_j).$$

Second identification: $u_1 \leftrightarrow \imath_2,\ u_2 \leftrightarrow \imath_1$. Again, a valid neutrosophic parallel connection is obtained with corresponding truth-membership, indeterminacy-membership, and falsity-membership values. Therefore, corresponding to each selected pair of deleted edges, exactly two possible neutrosophic parallel connections can be constructed.

Consequently, the total number of possible parallel connections is $2 \times mn = 2mn$. Hence, the number of possible parallel connections between the Min-max Neutrosophic Graphs $\widehat{\mathsf{G}}_1$ and $\widehat{\mathsf{G}}_2$ is $\boxed{2mn}$. □

Example 4.4. *Let $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ and $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$ be two Min-max Neutrosophic Graphs.*

Consider $\widehat{\mathsf{V}}_1 = \{a, b, c, d\}$ and $\widehat{\mathsf{V}}_2 = \{p, q, r, s\}$. The graph $\widehat{\mathsf{G}}_1$ is a square graph with edge set $\widehat{\mathsf{E}}_1 = \{ab, bc, cd, da\}$, while $\widehat{\mathsf{G}}_2$ is a star graph with edge set $\widehat{\mathsf{E}}_2 = \{pq, pr, ps\}$. The neutrosophic vertex values of $\widehat{\mathsf{G}}_1$ are

$$a = (0.9, 0.1, 0.1),\ b = (0.8, 0.2, 0.1),\ c = (0.7, 0.2, 0.2),\ \text{and } d = (0.6, 0.3, 0.2).$$

The neutrosophic edge values of $\widehat{\mathsf{G}}_1$ are

$$ab = (0.8, 0.2, 0.1),\ bc = (0.7, 0.2, 0.2),\ cd = (0.6, 0.3, 0.2),\ \text{and } da = (0.5, 0.3, 0.3).$$

Similarly, the neutrosophic vertex values of $\widehat{\mathsf{G}}_2$ are

$$p = (0.8, 0.1, 0.2),\ q = (0.7, 0.2, 0.2),\ r = (0.6, 0.2, 0.3),\ \text{and } s = (0.5, 0.3, 0.3).$$

The neutrosophic edge values of $\widehat{\mathsf{G}}_2$ are

$$pq = (0.7, 0.2, 0.2),\ pr = (0.6, 0.2, 0.3),\ \text{and } ps = (0.5, 0.3, 0.3).$$

Now, we choose the edges $\mathsf{e}_1 = ab \in \widehat{\mathsf{E}}_1$ and $\mathsf{e}_2 = pq \in \widehat{\mathsf{E}}_2$. Delete the edges ab and pq. Identify the vertices $a \leftrightarrow p$ to obtain a new vertex $\imath$. Then,

$$\widehat{\mathsf{T}}_\xi(\imath) = \widehat{\mathsf{T}}_\xi(a) \wedge \widehat{\mathsf{T}}_\xi(p) = \min\{0.9, 0.8\} = 0.8,$$
$$\widehat{\mathsf{I}}_\xi(\imath) = \widehat{\mathsf{I}}_\xi(a) \vee \widehat{\mathsf{I}}_\xi(p) = \max\{0.1, 0.1\} = 0.1,$$

and

$$\widehat{\mathsf{F}}_\xi(\imath) = \widehat{\mathsf{F}}_\xi(a) \vee \widehat{\mathsf{F}}_\xi(p) = \max\{0.1, 0.2\} = 0.2.$$

Hence,

$$\imath = (0.8, 0.1, 0.2).$$

Similarly, $b \leftrightarrow q$ to obtain another new vertex $\imath'$. Then,

$$\widehat{\mathsf{T}}_\xi(\imath') = \widehat{\mathsf{T}}_\xi(b) \wedge \widehat{\mathsf{T}}_\xi(q) = \min\{0.8, 0.7\} = 0.7,$$
$$\widehat{\mathsf{I}}_\xi(\imath') = \widehat{\mathsf{I}}_\xi(b) \vee \widehat{\mathsf{I}}_\xi(q) = \max\{0.2, 0.2\} = 0.2,$$

and

$$\widehat{\mathsf{F}}_\xi(\imath') = \widehat{\mathsf{F}}_\xi(b) \vee \widehat{\mathsf{F}}_\xi(q) = \max\{0.1, 0.2\} = 0.2.$$

Thus,

$$\imath' = (0.7, 0.2, 0.2).$$

Finally, introduce a new edge $\imath\imath'$ with neutrosophic values

$$\widehat{\mathsf{T}}_\chi(\imath\imath') = \widehat{\mathsf{T}}_\chi(ab) \wedge \widehat{\mathsf{T}}_\chi(pq) = \min\{0.8, 0.7\} = 0.7,$$
$$\widehat{\mathsf{I}}_\chi(\imath\imath') = \widehat{\mathsf{I}}_\chi(ab) \vee \widehat{\mathsf{I}}_\chi(pq) = \max\{0.2, 0.2\} = 0.2,$$

and

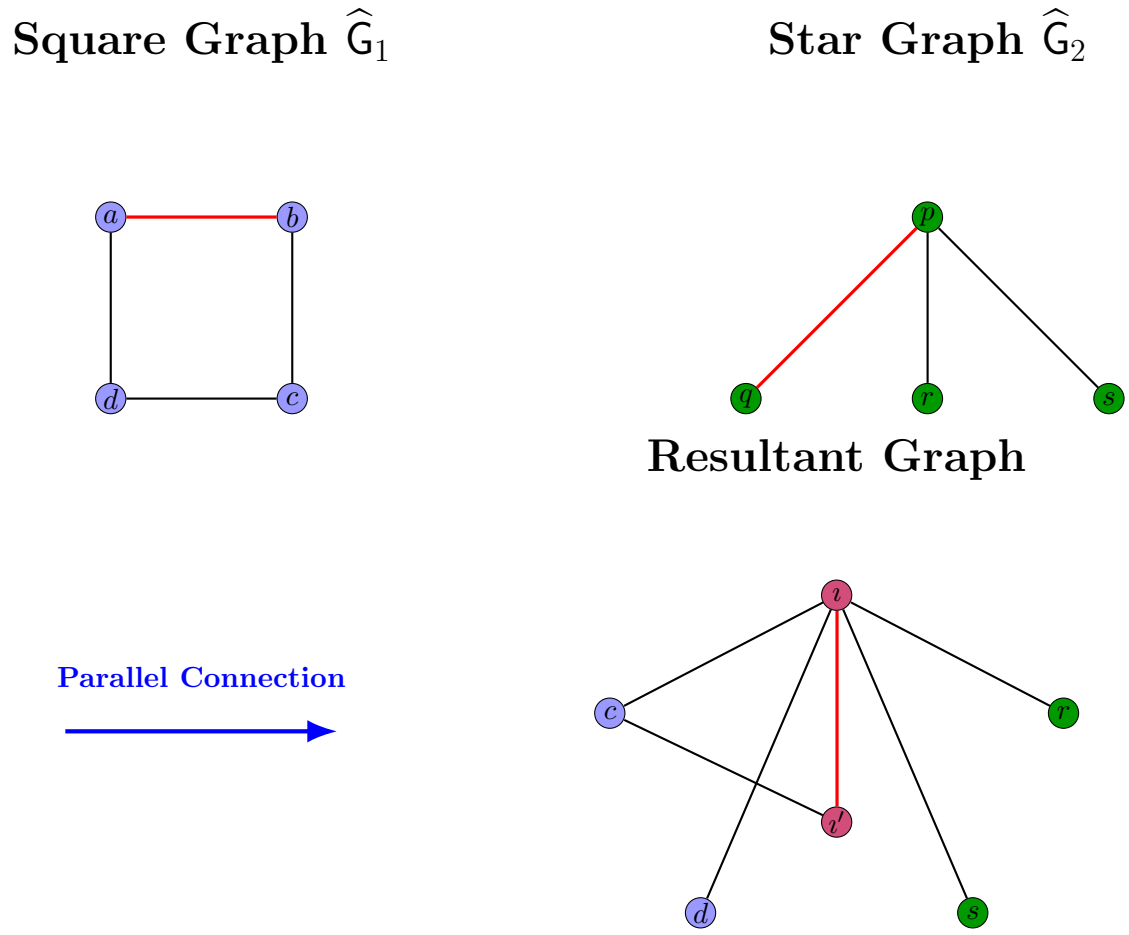
$$\widehat{\mathsf{F}}_\chi(\imath\imath') = \widehat{\mathsf{F}}_\chi(ab) \vee \widehat{\mathsf{F}}_\chi(pq) = \max\{0.1, 0.2\} = 0.2.$$

Therefore,

$$\imath\imath' = (0.7, 0.2, 0.2).$$

Hence, the parallel connection $P((\widehat{\mathsf{G}}_1, \mathsf{e}_1) : (\widehat{\mathsf{G}}_2, \mathsf{e}_2))$ *is obtained.*

The following figure illustrates the parallel connection of two Min-max Neutrosophic Graphs obtained by identifying the corresponding vertices after deleting the selected edges.



Theorem 4.3. *Let* $\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1)$ *and* $\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2)$ *be two Min-max Neutrosophic Graphs having* m *and* n *edges respectively. Then, the number of possible series connections between* $\widehat{\mathsf{G}}_1$ *and* $\widehat{\mathsf{G}}_2$ *is* $4mn$.

Proof. Suppose

$$|\widehat{\mathsf{E}}_1| = m \quad \text{and} \quad |\widehat{\mathsf{E}}_2| = n.$$

Let

$$\widehat{\mathsf{E}}_1 = \{\mathsf{e}_1, \mathsf{e}_2, \ldots, \mathsf{e}_m\}, \qquad \widehat{\mathsf{E}}_2 = \{\mathsf{e}'_1, \mathsf{e}'_2, \ldots, \mathsf{e}'_n\}.$$

To construct a series connection, one edge from each graph is selected and deleted. Hence, the number of ways of choosing one edge from $\widehat{\mathsf{G}}_1$ and one edge from $\widehat{\mathsf{G}}_2$ is mn. Let $\mathsf{e}_i = u_1u_2 \in \widehat{\mathsf{E}}_1$, $\mathsf{e}'_j = \imath_1\imath_2 \in \widehat{\mathsf{E}}_2$ be a chosen pair of edges. In a series connection, one vertex from each deleted edge is identified to form a single vertex. There are: $2 \times 2 = 4$ possible ways to choose one endpoint from e_i and one endpoint from e'_j:

Case 1:

$$u_1 \leftrightarrow \imath_1$$

Case 2:

$$u_1 \leftrightarrow \imath_2$$

Case 3:

$$u_2 \leftrightarrow \imath_1$$

Case 4:

$$u_2 \leftrightarrow \imath_2$$

Each identification produces a valid neutrosophic series connection, since the resulting vertex and edge memberships are defined using the Min-max neutrosophic operations:

$$\widehat{\mathsf{T}}_\xi = \min, \quad \widehat{\mathsf{I}}_\xi = \max, \quad \widehat{\mathsf{F}}_\xi = \max .$$

Thus, for each pair of edges, there are 4 possible series constructions.

Therefore, total number of series connections is $4 \times mn = 4mn$. Hence proved. □

Example of Series Connection in Graphs

Let

$$\widehat{\mathsf{G}}_1 = (\widehat{\mathsf{V}}_1, \widehat{\mathsf{E}}_1), \quad \widehat{\mathsf{V}}_1 = \{a, b, c\}, \quad \widehat{\mathsf{E}}_1 = \{ab, bc\}$$

and

$$\widehat{\mathsf{G}}_2 = (\widehat{\mathsf{V}}_2, \widehat{\mathsf{E}}_2), \quad \widehat{\mathsf{V}}_2 = \{\flat_1, \flat_2, z\}, \quad \widehat{\mathsf{E}}_2 = \{\flat_1\flat_2, \flat_2 z\}.$$

Both graphs are path graphs.

The number of edges are:

$$|\widehat{\mathsf{E}}_1| = 2, \quad |\widehat{\mathsf{E}}_2| = 2.$$

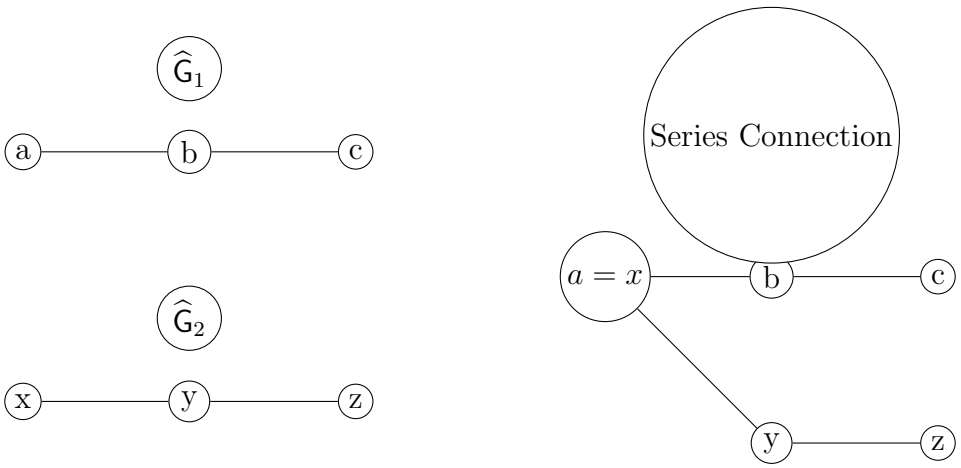
Thus, the number of possible series connections is:

$$4mn = 4 \times 2 \times 2 = 16.$$

A series connection is formed by:

(1) selecting one edge from each graph,
(2) deleting those edges,
(3) identifying one endpoint from each edge.

Example: Identify $a \equiv \flat_1$.



5. Conclusion

In this paper, we introduced and studied the concepts of parallel and series connections in Min-Max Neutrosophic Graphs. These operations extend classical graph-theoretic constructions into the neutrosophic environment, where uncertainty is modeled using truth, indeterminacy, and falsity membership functions. The structural behavior of these connections was analyzed, and exact combinatorial expressions were derived. It was shown that the number of possible parallel connections is $2mn$, while the number of series connections is $4mn$, where m and n are the number of edges in the respective graphs.

Furthermore, a new class of neutrosophic graphs, called Max-Max Neutrosophic Graphs, was introduced to broaden the framework of uncertainty-based graph modeling. The obtained results generalize existing work in fuzzy and intuitionistic fuzzy graphs and provide a unified combinatorial interpretation of graph connection operations under neutrosophic settings.

The proposed structures have potential applications in network theory, decision-making systems, and electrical circuit modeling, where uncertainty and indeterminacy play a significant role. Future work may focus on extending these operations to weighted neutrosophic graphs, hypergraphs, and dynamic network systems, as well as exploring algebraic and spectral properties of such constructions.

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